

Housing Tenure and Labor Market Self-Selection ^{*}

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Abstract

This paper presents a model that characterizes workers as having a greater incentive to select into homeownership when at a job with productivity that matches their ability. Differential search and mobility costs inherent across housing tenure result in homeownership serving as a signal for job match quality. We capitalize on a unique empirical platform comprising of the Danish Registry and present empirical evidence that provides support for the self-selection model and suggests that homeowners are indeed better matched than renters.

Keywords: Housing tenure, job match, search costs, labor market.

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1 Introduction

Does homeownership impose adverse effects on the labor market? This paper contributes to the highly debated externality question by presenting a self-selection model on housing and labor market decisions. Notably, the model predicts that workers self-select into homeownership when at a job with productivity that matches their ability, i.e. at a matched job. The suggestion that the decreased mobility of homeowners compromises their labor market outcomes - the Oswald hypothesis - has undergone intense examination over the last decade. While the initial tests of this hypothesis were of the aggregate (nation, state, province, etc.) relationship between unemployment rates and homeownership rates, more recent studies have concentrated on the differences between owners and renters outcomes. Many of these studies have motivated their tests using search models in the style of Pissarides (2000), using differences in either search ability, or the cost of unemployment as the trigger for the differences in outcomes. We build on a similar setup and depict that homeowners are more matched at their jobs than renters.

First, we present a search rational that models housing tenure across ownership or renting, and a choice of jobs with varying productivity levels. Central to our model is differential mobility costs across housing tenure. The model's primary prediction is that individuals take up home-ownership when at a matched job as the need for job related mobility is less. Interestingly, a stylized fact that homeowners stay at a job longer naturally flows from our model. However, we attribute such longer job durations for homeowners to the level of job match. Secondly, we characterize housing tenure change and predict a change to homeownership when the level of job match increases.

We capitalize on the richness of the data in the Danish Registry to test our model's prediction. Our empirical platform comprises of the working age population of Denmark

for the years 2008 to 2016 and enable us to track transitions across housing tenure as well as jobs. In a similar setup, Groes et al. (2015) find that occupational mobility is ‘U-shaped’, i.e. low and high wage earners within an occupation have a higher probability of separation. We build on this mobility pattern and show that homeowners lie at the center of the wage distribution, followed by renters at the tails. Intuitively, this implies that homeowners are more matched at their jobs than renters. Additionally, we track transitions to homeownership and show that renters change tenure when the level of job match increases. Thereby, we present cross-sectional and transitions related evidence for the self-selection model.

Our work relates to literature that connects labor market and housing tenure choices. Past work has extensively implied that reduced mobility due to homeownership constrains individuals’ labor market choices. For example, the ‘Oswald Hypothesis’ as explained in Oswald (1997) argues that homeownership has negative externalities on the labor market. In contrast, we present a theoretical basis and corresponding empirical evidence that suggest a more nuanced selection process across housing and labor markets.

Overall, this paper presents an optimal housing tenure based job match decision layered on to the classical investment-consumption trade-off characterized by Henderson and Ioannides (1983) and presents a self-selection mechanism where homeowners have a greater incentive to sort into matched jobs than renters. Additionally, empirical results using the Danish registry implies that housing tenure is an endogenous variable that signals the level of match between worker ability and job productivity. Furthermore, the model predictions are consistent with employment duration arguments documented by past literature.

The paper proceeds as follows. The next section describes the housing tenure based

model and assumptions. Section 3 presents the intuition behind the self-selection hypothesis, followed by corresponding empirical tests in Section 4. Finally, we discuss the implications of our work in Section 5 and conclude with in Section 6.

2 Background: Housing Tenure and the Labor Market

The literature originates with Dohmen (2005) and Munch et al. (2008). In these models, there are two locations, the job seeker’s current location, and “elsewhere”. Unemployed renters can accept jobs in either location at no cost, but homeowners who accept jobs elsewhere need to pay a moving cost. This cost represents the difficulty of selling a current home, or similar difficulties of disposing of, and acquiring a new, illiquid asset. For renters, the two locations are equivalent, and the reservation wage is the same for job offers in the two locations, while for owners the reservation wage for distant jobs is higher than that for renters, and lower for local jobs. The impact of homeownership on labor market outcomes is therefore ambiguous: assuming that the distribution of wage offers is the same from the two labor markets, the arrival rate of acceptable offers locally is going to be higher for owners than renters locally, but local for distant jobs. Depending on the two arrival rates, the exact form of the offer distribution and the size of the moving cost the unemployment rate for owners will be higher or lower. Furthermore, wages will be similarly higher or lower for owners, depending on the rate at which they accept distant and nearby jobs.

Coulson and Fisher (2009) provide two search models which extend these ideas. In one model, there is search and matching. The difference between owners and renters is

that renters are able to search in both locations, while owners can only search locally. Thus owners have a lower matching rate in their job search. When the employer and worker meet, they bargain (in a Nash bargaining framework) over the wage, and the firm, recognizing the higher cost to homeowners of walking away, offers a lower wage. On this account, owners have both a lower wage and higher unemployment than renters. Ringo (2014) adds to this basic framework the possibility of on-the-job search. Given the same basic constraints on the location of search, renters will have greater scope for finding new jobs. Therefore, renter employment spells will be shorter than owners. This is in contrast to all the aforementioned papers, who assume exogenous job detachment rates.

Coulson and Fisher (2009)'s second model posits that owners simply have a higher cost of unemployment. Firms post wage offers, which workers accept if it exceeds their reservation wage, which is lower for owners than for renters. A fraction of firms therefore offers the higher wage which is accepted by either type, and the rest offer the low wage which is accepted only by owners. Firms accordingly face a tradeoff and the fraction of firms that follow each strategy in equilibrium depends on the other parameters of the model. Owners have lower wages, but also lower unemployment because they have a greater probability of encountering an acceptable offer, similar to the mechanism in Munch et al. (2008). In effect, the difference between the two models turns on whether employers can observe the housing tenure choice of the prospective worker. If they can, they bargain down the wage accordingly, and if not, they post the wage blindly, so to speak.

These two models are therefore able to make predictions on the impact of the aggregate homeownership rate on the individual labor market outcome. In the bargaining model, a higher homeownership rate induces firm entry (through lower expected wages)

and therefore increases individual worker bargaining power, raising wages and lowering unemployment. In the posting model, a higher ownership rate causes a greater number of firms to offer the lower wage (because they are more likely to get accepted). Thus the average wage for owners falls, and the unemployment rate for renters rises.

Beugnot et al. (2014) combine a model of search and matching with an idea from Oswald (1997) that because of their lower utility flows while in unemployment, homeowners will not only accept lower wages but also longer commutes. Renters do not commute, but move to the new job location, wherever it is. The result of an increased homeownership rate will not only induce firm entry (improving all worker outcomes) but also reduce competition by decreasing the number of renters who compete for work at each location. Morescalchi (2015) and Yang (2015) endogenize search effort with respect to tenure type and location. Both show that when given the choice homeowners will have more intensive search in local areas. Morescalchi (2015) shows in his version of this model that owners will search more intensively in the local market and less so in the distant market, but that in sum there is less search on their part. They are therefore going to have slower exits out of unemployment—thus signing the effect which was ambiguous in Munch et al. (2008).

All of the aforementioned papers concentrate on frictions in the labor market that are caused by homeownership, but leave the housing market unmodeled. Advances in integrating the frictions that occur in the housing market itself began with Head and Lloyd-Ellis (2012) who (in effect) endogenize the cost of searching elsewhere by assuming that the length of time it takes to sell one's home (and thus be able to afford a house in the new location) depends on the strength of the local economy. On that account, the

ability to accept a distant job depends on the local labor market.¹

Head and Lloyd-Ellis (2012) opened a new dimension in the theoretical development of this topic in allowing tenure choice to be endogenous, in the sense that the households' constraints within either market dictate that choice. The model of De Graaff and Van Leuvensteijn (2013) has a similar feature. In the former case the (il)liquidity of the housing market is the major determinant, while in the latter, it is the households' draw from the wage distribution.

In our model we extend many of the dimensions of the housing tenure literature. In particular, we concentrate on match quality which we define as the match between the job requirements and the skills of the employee. As in Coulson and Fisher (2009), there are job postings by firms which arrive at workers. But all wages are identical; workers accept or reject the job on the basis of whether the job is within the acceptable 'distance' between the job and their own skills. Renters and owners establish a maximum level of mismatch. The tradeoff they face is that a greater level of acceptable mismatch increases the probability that a given offer is acceptable, but also increases the probability of job detachment in each period. Since renters have a lower cost of unemployment (again, a common assumption in this literature) they have a wider acceptable range. The predictions of the model are manifold, and some are new to this literature. An important one arises from the fact that owners are more restrictive about their job selection due to the potential of future detachments. Therefore the model predicts that owners will have both longer unemployment spells and longer employment spells.

¹Reed and Ume (2016) construct a similar theoretical model, but concentrate on the interaction between the housing and labor market in a single location. Therefore the issue of homeowner versus renter mobility does not arise.

3 The Model

In this section we develop a model which relates labor market outcomes to the decision to rent or buy. A worker is either a renter or a homeowner with ability a . The worker gets utility from consumption directly, and also from owning a house which we will treat as a consumption scalar M . There are also firms, which are each of type j . There is no saving in the model. A worker can either be working for a firm, or unemployed. If the worker is employed, she is paid her ability a regardless of the type of the firm. An unemployed worker gets a fraction of her employed income, Ba . Unemployed workers become employed with probability λ . Employed workers become separated with probability $S(a, j)$. That is, separation probability is a function of the worker's ability a and the firm type j . We will assume that the closer a and j , the lower the separation probability. We also assume that workers have period discount factor β .

The only decision a worker makes is whether to move from owning to renting. This can be done by paying cost F . Once a worker pays this cost, she receives the additional consumption scalar M every period she remains a home owner. We assume that employed home owners never lose their homes, but unemployed home owners lose their homes with probability δ . We intend this as a reduced form of mortgage defaults.

The Bellman Equations for home owners, employed and unemployed, are below. Homeowners make no decisions.

$$V_E^a(j, H) = u(Ma) + \beta ((s(j, a) ((1 - \delta)V_U^a(H) + \delta V_U^a(R))) + (1 - s(j, a))V_E^a(j, H)) \quad (1)$$

$$V_U^a(H) = (1 - \lambda) (u(Mba) + \beta ((1 - \delta)V_U^a(H) + \delta V_U^a(R))) + \lambda \mathbb{E} [V_E^a(j, H)] \quad (2)$$

The Bellman Equations for renters are below. Policy $k = 1$ means buying a home, and $k = 0$ means continuing to rent.

$$V_E^a(j, R) = \max_{k \in \{0,1\}} u(a - kF) + k\beta (s(j, a) ((1 - \delta)V_U^a(H) + \delta V_U^a(R)) + (1 - s(j, a))V_E^a(j, H)) \\ + (1 - k) \beta (s(j, a)V_U^a(R) + (1 - s(j, a))V_E^a(j, R)) \quad (3)$$

$$V_U^a(R) = (1 - \lambda) \left(\max_{k \in \{0,1\}} u(ba - kF) + k\beta V_U^a(H) + (1 - k) \beta V_U^a(R) \right) \\ + \lambda \mathbb{E} [V_E^a(j, R)] \quad (4)$$

The intuition in this model is that even though wages are the same at any firm, a worker will only buy a house when her job is stable. The reason is that unemployment carries with it a risk of losing the home. Thus the more stable the match, the more likely a worker is to make the decision to buy.

Proposition 1 *The relative benefit of buying a home is increasing in the stability of the match.*

The proof is in Appendix .

Although it is not the focus of our empirical exercise, with log utility higher ability (richer) workers will also be more likely to buy conditional on job stability. In a word, this is because of the curvature of utility, which causes buying a home to be more painful for low ability workers:

Corollary 1 *Conditional on job stability, if utility is log the relative benefit of buying a home is increasing in ability a .*

The proof is in Appendix

4 Empirical Implications

We turn to the Denmark Population Register to test the main predictions of the housing tenure model. The dataset comprises of annual information on socioeconomic variables of the population during the years 2008 to 2016. Using a similar dataset from the Denmark Statistics registry, Groes et al. (2015) find that occupational mobility is ‘U-shaped’ and directional. The U-shaped aspect implies that low and high wage earners within an occupation have a higher probability of separation, and the low (high) earners tend to move to new occupations with lower (higher) average wages. An interesting argument is presented where the wage of workers are shown to proxy for worker ability at the occupation.² We build on the occupational mobility framework documented in Groes et al. (2015) and relate it to housing tenure. The Danish dataset provides a unique advantage due the depth of job and occupation details for a large number of workers and thus enables us to characterize a measure of mismatch.

²Besides presenting empirical results on occupation mobility, Groes et al. (2015) provide a theoretical basis for observed wages serving as a proxy for ability.

First, we present a view of the dataset through summary statistics depicted in Table 1. After merging the housing and labor market datasets, our sample comprises of around 16.1 million wage-year observations. The mean hourly wage is 19.4 Danish krone and the mean monthly income is 26,370 krone. We consider working age individuals in the age range from 25 to 65 years and the average worker age in our sample is 43.9 years. Additionally, we present a frequency distribution of housing tenure and transitions in Table 2. The homeownership rate is 67.03%, along with 8.63% transitioning from renting to owning and 2.77% from owning to renting. We also present a view of the labor market transitions as measured by 30.35% changing jobs every year and 30.83% changing occupations, thereby indicating a high turnover rate.

Our primary empirical specification is based on the wage residual estimated from the following regression,

$$\log \hat{wage} = f(\text{occupation f.e.}, \text{year f.e.}) + \hat{\epsilon} \quad (5)$$

Here, the predicted residual, $\hat{\epsilon}$, characterizes an individual's 'mismatch' at the occupation. Note that we are interested in positive as well as negative residuals, i.e. if the mismatch is on either side of the wage distribution. Our objective is to measure the level of mismatch for an individual at the current job based on his/her relative position in the wage distribution. Table 3 presents the summary statistics corresponding to the predicted residuals and absolute residuals. The standard deviation of the residual is 0.31 and presents a source of variation across the level of job match-ness. We also depict the difference in absolute residuals of individuals across consecutive years as this measures the change in the level of job mismatch.

Next, we present two stylized facts as documented by past work to depict the cov-

erage of our empirical setup. First, Table 4 presents the relation between job duration and the wage residual. We see shorter job durations as the level of mismatch increases for positive and negative residuals, implying a greater likelihood of detachment when at a mismatched job.³ Additionally, we present the relation between housing tenure and job duration in Table 5. We document that homeowners have longer job durations when mismatched at either end of the wage distribution.

We now present our primary result through a cross-sectional regression relating housing tenure and wage mismatch. We estimate the following specification,

$$Owner = \beta_1 abs(\hat{\epsilon}) + \beta controls + \lambda \tag{6}$$

Owner is a binary variable indicating homeownership, $abs(\hat{\epsilon})$ is the absolute predicted residual from equation (5), or for our purposes, a measure of job mismatch. We include controls such as: log annual income or highest and lowest income decile indicators, Age and Age square, number of children, family structure type, year and municipality fixed effects. Table 6 presents the cross-sectional results relating to equation (6). Note that across all specifications, the coefficient of $abs(\hat{\epsilon})$ is negative and statistically significant at the 1% level indicating a lower likelihood of homeownership when the level of mismatch increases. The variables *ln_income* and highest, lowest decile income indicators are introduced to account for the affordability issue, i.e. homeownership is more affordable to folks with higher wages. Next, Table 7 presents a similar analysis but depicts the regression results for positive and negative residuals separately, thereby characterizing homeowners as having better matches.

³This is similar to the Groes et al. (2015) argument.

We turn our attention to transitions to homeownership. We estimate the following specification,

$$\Delta R_{to-O} = \beta_1 \Delta abs(\hat{\epsilon}) + \beta \text{ controls} + \lambda \quad (7)$$

Here, ΔR_{to-O} is an indicator characterizing transitions to homeownership and $\Delta abs(\hat{\epsilon})$ measures the difference in the absolute residual across consecutive years. We also include controls for job, occupation and location change. Table 8 presents the estimated regressions across several specifications. The base specification without any controls and the specification with the income deciles as controls have a statistically significant negative coefficient for $\Delta abs(\hat{\epsilon})$ and implies that renters change housing tenure to homeownership when the level of job match increases. Thus, we present support for our transition related predictions on housing tenure.

5 Discussion

Relate our findings to the broader housing tenure and labor literature.

Implications for future work

6 Concluding Remarks

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Table 1: Summary Statistics

	mean	sd	count
Wage	19.42	1.45	18,201,336
Income	26,370	1.80	18,201,336
Age	43.90	10.57	18,201,336

Table 2: Frequencies

Variable	N	Percent
Male	18,201,336	50.60
Owner	18,201,336	7.03
Job Change	18,201,336	30.35
Occ Change	18,201,336	30.83
R to O	4,581,655	8.63
O to R	9,969,731	2.77

Table 3: Summary Statistics - Residuals

	mean	sd	count
Residual	0.00	0.31	16,108,168
Absolute Residual	0.21	0.23	16,108,168
Annual Change in Residual	0.00	0.22	12,835,192

Table 4: Regression relating job duration and residuals.

	(1)		(2)	
	Negative Residual		Positive Residual	
	b	se	b	se
residual	108.402***	(2.2256)	-96.678***	(2.0852)
_cons	668.781***	(0.6930)	685.302***	(0.8202)
r2	0.002		0.003	
N	1,031,124		810,833	

Table 5: Regression relating job duration and homeownership.

	(1)		(2)	
	Negative Residual		Positive Residual	
	b	se	b	se
owner	72.751***	(1.0232)	136.744***	(1.6934)
_cons	598.817***	(0.8033)	785.124***	(1.3789)
r2	0.005		0.006	
N	1,031,124		1,061,602	

Table 6: Main CS regression relating homeownership to residuals.

	(1)		(2)		(3)	
	owner		owner		owner	
	b	se	b	se	b	se
Absolute Residual	-0.050***	(0.0005)	-0.003***	(0.0004)	-0.039***	(0.0005)
ln_income			0.101***	(0.0002)		
Age			0.015***	(0.0001)	0.018***	(0.0001)
Age square			-0.000***	(0.0000)	-0.000***	(0.0000)
# Children			0.018***	(0.0001)	0.018***	(0.0001)
Highest Income decile					0.129***	(0.0003)
Lowest Income decile					-0.086***	(0.0004)
_cons	0.681***	(0.0002)	-0.994***	(0.0025)	-0.019***	(0.0020)
Year f.e.	No		Yes		Yes	
Municipality f.e.	No		Yes		Yes	
Family Structure f.e.	No		Yes		Yes	
r2	0.001		0.316		0.312	
N	16,108,169		16,108,169		16,108,169	

Table 7: Regression relating homeownership and positive, negative residuals.

	(1)		(2)		(3)		(4)	
	Negative Residual	Positive Residual	Negative Residual	Positive Residual	Negative Residual	Positive Residual	Negative Residual	Positive Residual
	b	se	b	se	b	se	b	se
Residual	0.005***	(0.0007)	-0.019***	(0.0006)	0.045***	(0.0007)	-0.036***	(0.0006)
ln_income	0.089***	(0.0003)	0.104***	(0.0003)				
Age	0.014***	(0.0001)	0.015***	(0.0001)	0.016***	(0.0001)	0.018***	(0.0001)
Age square	-0.000***	(0.0000)	-0.000***	(0.0000)	-0.000***	(0.0000)	-0.000***	(0.0000)
# Children	0.014***	(0.0002)	0.022***	(0.0002)	0.014***	(0.0002)	0.022***	(0.0002)
Highest Income decile					0.091***	(0.0017)	0.111***	(0.0004)
Lowest Income decile					-0.077***	(0.0005)	-0.097***	(0.0007)
_cons	-0.819***	(0.0038)	-1.021***	(0.0040)	0.039***	(0.0026)	-0.017***	(0.0032)
Year f.e.	Yes		Yes		Yes		Yes	
Municipality f.e.	Yes		Yes		Yes		Yes	
Family structure f.e.	Yes		Yes		Yes		Yes	
r2	0.331		0.288		0.326		0.285	
N	8954647.000		7153522.000		8954647.000		7153522.000	

Table 8: Transitions from R to O - All Observations

	(1)		(2)		(3)	
	owner_chg		owner_chg		owner_chg	
	b	se	b	se	b	se
Change in Abs Residual	-0.002**	(0.0006)	0.000	(0.0006)	-0.005***	(0.0006)
ln_income			0.035***	(0.0002)		
Age			-0.003***	(0.0001)	-0.002***	(0.0001)
Age square			0.000***	(0.0000)	-0.000	(0.0000)
# Children			0.007***	(0.0002)	0.006***	(0.0002)
Highest Income decile					0.054***	(0.0006)
Lowest Income decile					-0.030***	(0.0004)
_cons	0.087***	(0.0001)	-0.208***	(0.0030)	0.134***	(0.0023)
Year f.e.	No		Yes		Yes	
Municipality f.e.	No		Yes		Yes	
Family Structure f.e.	No		Yes		Yes	
Location Change f.e.	No		Yes		Yes	
Job Change f.e.	No		Yes		Yes	
Occupation Change f.e.	No		Yes		Yes	
r2	0.000		0.157		0.156	
N	4052843.000		4052843.000		4052843.000	

Appendix

7 Proof of proposition 1

The benefit of buying a home is increasing in the stability of the match.

Proof: Consider a renter of ability a employed at a firm of type j . Her relative benefit of buying a home compared with remaining a renter NB is:

$$\begin{aligned}
 NB &= u(a - F) + \beta (s(j, a) ((1 - \delta)V_U^a(H) + \delta V_U^a(R)) + (1 - s(j, a))V_E^a(j, H)) \\
 &\quad - u(a) - \beta (s(j, a)V_U^a(R) + (1 - s(j, a))V_E^a(j, R)) \\
 &= u(a - F) - u(a) + \beta (s(j, a) ((1 - \delta) (V_U^a(H) - V_U^a(R)))) \\
 &\quad + \beta ((1 - s(j, a)) (V_E^a(j, H) - V_E^a(j, R)))
 \end{aligned}$$

This expression involves the value of remaining a renter $V_E^a(j, R)$. Since this value depends on the policy of buying or not, it is convenient to split the proof into two cases. The first is such that $NB > 0$, so that the policy will be buying. The second is when $NB < 0$ so that the policy will be to not buy.

Case 1: $NB > 0$. In this case, the only difference between $V_E^a(j, H)$ and $V_E^a(j, R)$ is that consumption is $u(a - F)$ when a renter, and $u(Ma)$ when a home owner. Thus we can rewrite:

$$\begin{aligned}
 NB &= u(a - F) - u(a) + \beta (s(j, a) ((1 - \delta) (V_U^a(H) - V_U^a(R)))) \\
 &\quad + \beta ((1 - s(j, a)) (u(Ma) - u(a - F)))
 \end{aligned}$$

Taking the derivative with respect to job type in this case:

$$\begin{aligned}
 \frac{\partial NB}{\partial j} &= \frac{\partial s}{\partial j} \beta ((1 - \delta) (V_U^a(H) - V_U^a(R)) - (u(Ma) - u(a - F))) \\
 \frac{\frac{\partial NB}{\partial j}}{\frac{\partial s}{\partial j}} &= \beta ((1 - \delta) (V_U^a(H) - V_U^a(R)) - (V_E^a(j, H) - V_E^a(j, R))) \\
 &\leq \beta ((1 - \delta) (V_E^a(j, H) - V_U^a(R)) - (V_E^a(j, H) - V_U^a(R))) \\
 &= -\delta \beta (V_E^a(j, H) - V_U^a(R)) < 0
 \end{aligned}$$

The substitutions on the second line follow from our assumption that workers prefer any job over unemployment.⁴ Here we have shown that if the stability of a match increases ($\frac{\partial s}{\partial j} < 0$), then the benefit of purchasing a home increases. Since the left hand side is less than zero, it must be that $\frac{\partial NB}{\partial j}$ and $\frac{\partial s}{\partial j}$ must have opposite signs. In particular, if $\frac{\partial s}{\partial j} < 0$,

⁴In a more general model, revealed preference would also work. We only observe a worker in a job if that worker prefers the job to unemployment.

so that job stability increases, then $\frac{\partial s}{\partial j} > 0$, the relative benefit of buying increases. This is what we are trying to prove.

Case 2: $NB < 0$. The argument is similar to Case 1. In this case, the difference between $V_E^a(j, H)$ and $V_E^a(j, R)$ is $NB + u(Ma) - u(a - F)$. We can rewrite:

$$NB = u(a - F) - u(a) + \beta (s(j, a) ((1 - \delta) (V_U^a(H) - V_U^a(R)))) \\ + \beta ((1 - s(j, a)) (NB + u(Ma) - u(a - F)))$$

Taking the derivative with respect to job type in this case:

$$\begin{aligned} \frac{\partial NB}{\partial j} &= \frac{\partial s}{\partial j} \beta ((1 - \delta) (V_U^a(H) - V_U^a(R)) - (NB + u(Ma) - u(a - F))) + \beta (1 - s) \frac{\partial NB}{\partial j} \\ \frac{\frac{\partial NB}{\partial j}}{\frac{\partial s}{\partial j}} &= \frac{\beta}{1 - \beta(1 - s)} ((1 - \delta) (V_U^a(H) - V_U^a(R)) - (V_E^a(j, H) - V_E^a(j, R))) \\ &\leq \frac{\beta}{1 - \beta(1 - s)} ((1 - \delta) (V_E^a(j, H) - V_U^a(R)) - (V_E^a(j, H) - V_U^a(R))) \\ &= -\delta \frac{\beta}{1 - \beta(1 - s)} (V_E^a(j, H) - V_U^a(R)) < 0 \end{aligned}$$

This ends the proof.

8 Proof of Corollary 1

Conditional on job stability, if utility is log the relative benefit of buying a home is increasing in ability a .

Proof: Consider a renter of ability a employed at a firm of type j . Her relative benefit of buying a home compared with remaining a renter NB is:

$$NB = \ln(a - F) + \beta (s ((1 - \delta) V_U^a(H) + \delta V_U^a(R)) + (1 - s) V_E^a(j, H)) \\ - \ln(a) - \beta (s V_U^a(R) + (1 - s) V_E^a(j, R)) \\ = \ln(a - F) - \ln(a) + \beta (s ((1 - \delta) (V_U^a(H) - V_U^a(R)))) \quad (8)$$

$$+ \beta ((1 - s) (V_E^a(j, H) - V_E^a(j, R))) \quad (9)$$

The general insight in this proof is that with log utility, there are only two possible derivatives of period utility with respect to a . If period utility is $\ln(a - F)$, then the derivative is $\frac{1}{a - F}$. If the period utility is $\ln(Ca)$ for any constant, then the derivative is $\frac{1}{a}$. Moreover, this difference in period utilities can only be maintained for a single period, because the act of buying moves a renter to the homeowner value function. Thus the minimum possible value of $\frac{\partial V_E^a(j, H)}{\partial a} - \frac{\partial V_E^a(j, R)}{\partial a} = \frac{1}{a} - \frac{1}{a - F}$. The difference between the value

of being unemployed is more complicated, because it involves an expectation. The same argument goes through, however, since state by state the minimum difference between period utility is $\frac{1}{a} - \frac{1}{a-F}$. Thus, again the minimum difference $\frac{\partial V_U^a(H)}{\partial a} - \frac{\partial V_U^a(R)}{\partial a} = \frac{1}{a} - \frac{1}{a-F}$. We want to show that $\frac{\partial NB}{\partial a} > 0$.

Taking the derivative of NB with respect to ability:

$$\begin{aligned} \frac{\partial NB}{\partial a} &= \frac{1}{a-F} - \frac{1}{a} + \beta \left(s \left((1-\delta) \left(\frac{\partial V_U^a(H)}{\partial a} - \frac{\partial V_U^a(R)}{\partial a} \right) \right) \right) \\ &\quad + \beta \left((1-s) \left(\frac{\partial V_E^a(j,H)}{\partial a} - \frac{\partial V_E^a(j,R)}{\partial a} \right) \right) \end{aligned}$$

From our argument above:

$$\begin{aligned} \frac{\partial NB}{\partial a} &> \frac{1}{a-F} - \frac{1}{a} + \beta \left(s \left((1-\delta) \left(\frac{1}{a} - \frac{1}{a-F} \right) \right) \right) \\ &\quad + \beta \left((1-s) \left(\frac{1}{a} - \frac{1}{a-F} \right) \right) \\ &= (1 - \beta(s(1-\delta) + (1-s))) \left(\frac{1}{a-F} - \frac{1}{a} \right) > 0 \end{aligned}$$

This concludes the proof.

9 Model simulation

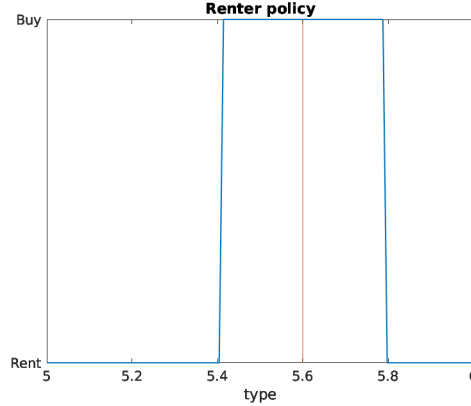
In this section we simulate the model developed in Section 3. We assume log utility with an offset of F :

$$u(x) = \ln(F + x)$$

We offset the utility with the cost of purchasing a house F , so that the utility of purchasing a home is always well-defined. That is, even if income is very low, the argument of the utility function in the case of purchasing a home will never be negative. We assume that the separation probability is given by:

$$s(j, a) = (a - j)^2$$

Figure 1: Employed renter policy function



We assume that wage offers are drawn from a uniform distribution with support $[5, 6]$, and worker quality is drawn from a uniform distribution with support $[5.2, 5.8]$. We also need to specify a number of parameters:

Parameter	Value	Definition
β	0.95	discount factor
M	1.2	consumption scalar of owning
λ	0.1	job finding prob.
B	0.6	unemployment replacement rate
F	10.3	cost of buying
δ	0.05	prob of losing house if unemp.

With this setup, the maximum mismatch between worker and firm $|a - j|$ is 0.8. In our simulation, the average mismatch of homeowners $|a - j|$ is 0.305, and the average mismatch of renters is 0.334. That is, homeowners have less mismatch in the cross section. Unemployed renters do not buy. Figure 1 contains the policy function for employed renters of ability 5.6 in the model. Type on the x-axis refers to job type. As expected, renters only buy if mismatch is low enough. Figure 2 contains similar information presented another way. The two lines are the value to an employed renter of ability 5.6 of continuing to rent, and to buy. The value of buying is higher than the value of renting when mismatch is low enough.

Figure 2: Comparison of value of buying vs renting for employed renters

